On-line identification of systems with delayed inputs

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***Abstract*— This communication deals with on-line identification of systems with delayed inputs. It is based on new non-asymptotic algebraic estimation techniques. A concrete case-study and an application to transmission delays are discussed. Several successful numerical simulations are provided even with noisy data.**

I. INTRODUCTION

The delay phenomenon constitutes one of the major complexity components of networked control, since actuators, sensors, computers, field networks and wireless communications that are involved in feedback loops unavoidably introduce dead-times, which might even be time-dependent. Despite numerous advances in this field, delay remains a theoretical and practical challenge (see, e.g., the survey [12]) for systems controlled over networks. Among the numerous open problems, the on-line delay identification is most crucial. On the one hand, various powerful control techniques (predictors, flatness-based predictive control, finite spectrum assignments, observers, ...) may be applied if the dead-time is known. On the other hand, the existing identification techniques for time-delay systems (see, e.g., [11] for a modified least squares technique, and a survey in [2]) generally suffer from poor speed performance. This communication is a first step towards a delay-adaptation of the fast identification techniques that were recently proposed [9] for linear, finite-dimensional models. Let us recall that those techniques are not asymptotic, and do not need any statistical knowledge of the noises corrupting the data. (See, e.g., [10] for applications to nonlinear state estimation, [5], [6] for linear and nonlinear diagnosis, and [8] for signal processing. Several successful laboratory experiments have already been performed; see, e.g., [3].) A concrete case-study and a transmission delay are illustrating our results and demonstrating their robustness with respect to noisy data. See [14] for another most interesting application.

We adopt in this paper a distributional formulation (compare with [9]) from which the parameters as well as the input delays may be easily estimated. The identification procedure employs elementary input signals, i.e., piecewise constant or polynomial time functions.

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II. MATHEMATICAL FRAMEWORK

A. Notations

We recall here some standard definitions and results from distribution theory [13] and fix the notations we shall use in the sequel. Let Ω be an open subset of $\mathbb{R}$. The space of C^∞-functions having compact support in Ω is denoted by $\mathscr{D}(\Omega)$, $\mathscr{D}'(\Omega)$ is the space of distributions on Ω, i.e., the space of continuous linear functionals on $\mathscr{D}(\Omega)$. The complement of the largest open subset of Ω in which a distribution T vanishes is called the support of T and is written $\operatorname{supp} T$. Write $\mathscr{D}'_+$ (resp. $\mathscr{E}'$) the space of distributions with support contained in $[0,\infty)$ (resp. compact support). It is an algebra with respect to convolution with identity δ, the Dirac distribution. When concentrated at a point $\{\tau\}$, the latter distribution $\delta(t-\tau)$ is written δ_τ. A distribution is said to be of order r if it acts continuously on C^r-functions but not on C^{r-1}-functions. Measures and functions are of order 0. Functions are considered through the distributions they define and are therefore indefinitely differentiable. If y is a continuous function except at a point a with a finite jump σ_a, its derivative dy/dt is $dy/dt = \dot{y} + \sigma_a \delta_a$, where $\dot{y}$ is the distribution stemming from the usual derivative of y. Derivation, integration and translation can be formed from the convolution products $\dot{y} = \delta^{(1)} * y$, $\int y = H * y$, $y(t-\tau) = \delta_\tau * y$, where $\delta^{(1)}$ is the derivative of the Dirac distribution, and H is the familiar Heaviside function. With a slight abuse of notations, we shall write $H^k y$ the k^{th}-order iterated integration of y and, more generally, T^k the iterated convolution product of order k. For $S, T \in \mathscr{D}'_+$, $\operatorname{supp} S * T \subset \operatorname{supp} S + \operatorname{supp} T$, where the sum in the right hand side is defined by $\{x+y;\, x \in \operatorname{supp} S,\ y \in \operatorname{supp} T\}$. Finally, with no danger of confusion, we shall sometimes denote $T(s)$, $s \in \mathbb{C}$, the Laplace transform of T.

B. Background

Multiplication of two distributions (say α and T) is not always defined. This operation makes however sense if one of the two terms is a smooth function. Several properties can be derived from such products. The most important ones here are given bellow. In particular, the next Theorem is the key result from which most of the parameters (including the delays) can be estimated.

Theorem 1: If T has a compact support K and is of finite order m, $\alpha T = 0$ whenever α and all its derivatives of order $\leq m$ vanish on K [13].
The following examples illustrate this statement when α is a polynomial function, and T a singular distribution. Note

that, in forming the product αT, the delay τ involved in the argument $T(t-\tau)$ now appears also as a coefficient.

$$
\begin{gathered}
t\,\delta = 0, \\
(t-\tau)\,\delta_\tau = 0, \\
t^2(t-\tau)\,(a\,\delta^{(1)} + b\,\delta_\tau) = 0
\end{gathered}
$$

The usual Leibniz rule $(\alpha T)' = \alpha' T + \alpha T'$ remains valid. Thanks to Theorem 1 the statement for the product $t\,\delta$ can be extended to $t^l\,\delta^{(n)} = 0$ for $l > n$, and

$$t^l\,\delta^{(n)} = (-1)^l \frac{n!}{(n-l)!}\,\delta^{(n-l)}, \quad l \le n \tag{1}$$

We shall make use of another property involving both multiplication by t^n and the convolution product, in case one of the two distributions (S or T) is of compact support:

$$t^n\,(S*T) = \sum_{k=0}^{n} C_n^k\,(t^k S)*(t^{n-k}\,T) \tag{2}$$

The C_n^k are the familiar binomial coefficients. Combining rules (1) and (2) with $S = \delta^{(p)}$ and $T = y$ allows us to transform terms of the form $t^n y^{(p)}$ into linear derivatives sums of products $t^k y$. Setting $z_i = t^i y$ yields

$$t^3 y^{(2)} = t^3\,(\delta^{(2)} * y) = -6z_1 + 6z_2^{(1)} - z_3^{(2)} \tag{3}$$

Note that integrating twice this expression by considering $H^2 t^3 y^{(2)}$ results in integration by parts formulae.

III. APPLICATION

A. Identification

Consider a first order system with a delayed input[1]:

$$\dot{y} + a\,y = y(0)\,\delta + \gamma_0\,H + b\,u(t-\tau) \tag{4}$$

where γ_0 is a constant perturbation, a, b, and τ are constant parameters. The coefficient a is assumed to be known for the moment. Consider also a step input $u = u_0 H$. A first order derivation yields

$$\ddot{y} + a\dot{y} = \varphi_0 + \gamma_0\,\delta + b\,u_0\,\delta_\tau \tag{5}$$

where $\varphi_0 = (\dot{y}(0) + ay(0))\,\delta + y(0)\,\delta^{(1)}$, of order 1 and support $\{0\}$, contains the contributions of the initial conditions. By Theorem 1, multiplication by a function α such that $\alpha(0) = \alpha'(0) = 0$, $\alpha(\tau) = 0$ yields interesting simplifications. Set $\alpha(t) = t^3 - \tau t^2$:

$$
\begin{aligned}
t^3\,[\ddot{y} + a\dot{y}] &= \tau t^2\,[\ddot{y} + a\dot{y}] &\quad (6)\\
bu_0\,t^3\delta_\tau &= bu_0\,\tau t^2 \delta_\tau &\quad (7)
\end{aligned}
$$

The delay τ becomes available after $k \ge 1$ successive integrations. More precisely, since $\mathrm{supp}\,H^k\delta_\tau \subset (\tau,\infty)$, equation (7) shows that all the obtained functions will vanish on $(0,\tau)$ and the delay is consequently not identifiable on this interval. Conversely, those functions being nonzero for all $t > \tau$, the delay is everywhere identifiable on (τ,∞). We therefore obtain from (6):

$$\tau = \frac{H^k(w_0 + a\,w_3)}{H^k(w_1 + a\,w_2)}, \qquad t > \tau \tag{8}$$

where, by virtue of equation (2) and recalling the notation $z_i = t^i y$ of the previous Section, we set:

$$
\begin{aligned}
w_0 &= t^3 y^{(2)} = -6z_1 + 6z_2^{(1)} - z_3^{(2)} \\
w_1 &= t^2 y^{(2)} = -2z_0 + 4z_1^{(1)} - z_2^{(2)} \\
w_2 &= t^2 y^{(1)} = 2z_1 - z_2^{(1)} \\
w_3 &= t^3 y^{(1)} = 3z_2 - z_3^{(1)}
\end{aligned}
$$

These coefficients show that $k \ge 2$ integrations are avoiding any derivation in the delay identification. Figure 1 is showing a partial realization scheme of the terms involved in (8). See Figure 2 for a numerical simulation with $k = 2$ integrations, and where $y(0) = 0.3, a = 2, \tau = 0.6, \gamma_0 = 2, b = 1, u_0 = 1$.

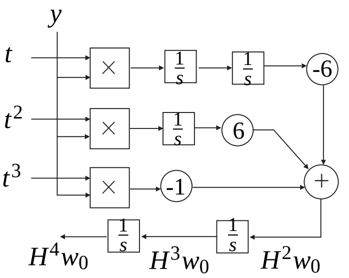

Fig. 1. Realization scheme of $H^k w_0$

Due to the non identifiability on $(0,\tau)$, the delay τ is set to zero until the numerator or denominator in the right hand side of (8) reaches a significant nonzero value.

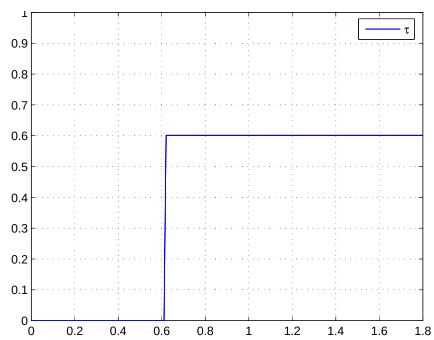

Fig. 2. Delay identification from (8)

The algorithm given in (8) only requires the values of a and of the output y. If a is also unknown, the same approach may be utilized for a simultaneous identification. The following relation is easily derived from (6):

$$\tau(H^k w_1) + a\,\tau(H^k w_2) - a\,(H^k w_3) = H^k w_0 \tag{9}$$

A linear system with unknown parameters $(\tau, a\tau, a)$ is obtained by using different integration orders

[1] Such systems, where the delays only appear in the control variables, are most common in practice. See [7] for their theoretical background, and their control.

$$\begin{pmatrix} H^2 w_1 & H^2 w_2 & H^2 w_3 \\ H^3 w_1 & H^3 w_2 & H^3 w_3 \\ H^4 w_1 & H^4 w_2 & H^4 w_3 \end{pmatrix} \begin{pmatrix} \hat{\tau} \\ \hat{a\tau} \\ -\hat{a} \end{pmatrix} = \begin{pmatrix} H^2 w_0 \\ H^3 w_0 \\ H^4 w_0 \end{pmatrix}$$

The resulting numerical simulations are shown in Figure 3. For the previous identifiability reason, the obtained linear system may be not consistent for $t < \tau$. Moreover, and unlike the single delay case, a local loss of identifiability (see [1]) may occur for $t > \tau$ as suggested in Figure 3 for $t \approx 1.5$s.

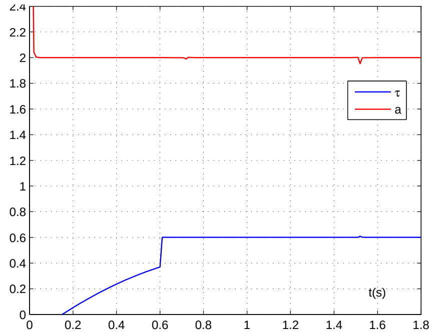

Fig. 3. Simultaneous identification from (9)

Multiplying (5) by other C^∞-functions permit the estimation of other parameter combinations. By setting, for instance, $\alpha(t) = t(t-\tau)$ and by using the same previous techniques we obtain

$$H^k t(\ddot{y} + a\dot{y})\,\tau + H^k\,\tau y(0) = H^k t^2(\ddot{y} + a\dot{y})$$

from which both delay and initial condition may be identified. The only coefficient for which the explicit value of τ is required is its associated parameter b. Finally, due to the fast convergence of the algorithms, we may also consider a separate procedure in which the undelayed terms are firstly identified and used again for the delay identification.

Remark 1: In case of step inputs with a time-varying delay $\tau(t)$, the algorithm (8) may still converge to a fixed value. Note that in this case, the contribution of the input in (4) may be written $bu_0 H \circ \varphi$, $\varphi(t) = t - \tau(t)$, where $\circ$ denotes the composition of functions. If $\varphi(t_0) = 0$, $\varphi'(t_0) \neq 0$, and provided some smoothness assumptions on $\tau(t)$, we obtain

$$(H \circ \varphi)' = \varphi' H' \circ \varphi = \varphi' \delta \circ \varphi = (\varphi'/\varphi'(t_0))\,\delta_{t_0} = \delta_{t_0}$$

The algorithm is converging to t_0. Although step inputs are clearly inadequate for the identification of time-varying delays, constant delays are, in some sense, particular cases of the present approach.

B. Robustness

As mentioned in Section II-B, iterated convolutions using H^k result in nothing but integration by part formulae. With noisy data, integration with $H(s) = 1/s$ may be replaced by any strictly proper transfer function and particularly by low pass filters such as $T(s) = 1/(\gamma s + 1)$. The following simulations show favorable robustness properties of the proposed delay identifier with respect to noise corrupted data. The input and output perturbation noises powers were taken to be of amplitude 2×10^{-6} and 4×10^{-6} respectively, while $\gamma = 2$ was used for the filter.

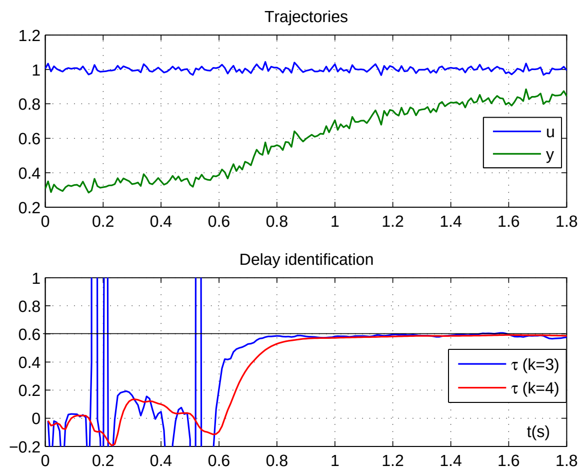

Fig. 4. Delay identification with noise in the data

Remark 2: See [4] for a nonstandard analysis of noises which explains the above robustness results.

Remark 3: Another possible explanation of the convergence may be deduced via the power spectrum of the additional terms that appear in (8) when the output y is replaced by a measurement $m = y + \nu$. The contribution of the noise ν is made through terms of the form $T^k((t^p\nu)^{(q)})$ for which the power spectrum is given by

$$|T(j\omega)|^k (j\omega)^q \frac{d^p \phi_{\nu\nu}(\omega)}{d\omega^p}.$$

If the noise spectrum $\phi_{\nu\nu}(\omega)$ is assumed to be constant on a sufficiently large strip $-L < \omega < L$ (which is the case for white noise approximations), and by virtue of the strict causality obtained with $k > q$, these quantities, and hence the noise effects remain negligible.

IV. APPLICATION TO TRANSMISSION DELAYS

The input delay of the previous Section has been estimated because it also corresponded to the commutation instant of the right hand side $\gamma_0 H + bu_0 H(t-\tau)$ of (4). Based on this observation, our techniques may be extended to a possible infinite number K of delays by means of one of the three following approaches:

1) A multiplication with the C^∞-function $t^2(t - h_1)\cdots(t - h_K)$, if K is finite,
2) a recursive identification,
3) a local identification if one assumes a lower bound for two successive delays, i.e., $h_{k+1} - h_k > \Delta$.

The first case may however lead to a large size linear system for which the delays remain unknown until $t > h_K$, while error propagations may result from the second case. We next consider the next case.

Assume that a discrete reference signal $\{u_k\}$ with fixed period T is sent from a Master to a continuous process (Slave) for which each data is hold until the next step. Due to the transmission line, the actual input of the process will consist in a piecewise constant signal of the form

$$u_a = \sum_{k=0}^{\infty} u_k \chi_k$$

where χ_k denotes the characteristic function of the interval $[kT+\tau_k, (k+1)T+\tau_{k+1}]$. Based on the output observations, we propose an on-line identification scheme of the time-instants $kT+\tau_k$. By this way, if the period T is known and if Master and Slave share the same time origin, then the transmission delays τ_k become available on-line from the only measurements of y. For simplicity reasons, the process under consideration consists in a second order linear system

$$\ddot{y} + a_1 \dot{y} + a_0 y = \varphi_0 + \gamma_0 H + b u_a$$

where $\gamma_0 H$ is a constant perturbation, and φ_0 (of order 1 and support $\{0\}$) contains the initial conditions. Following the previous section, derivation and multiplication by $t^3(t-\lambda)$ yields

$$t^3(t-\lambda)(y^{(3)} + a_1 y^{(2)} + a_0 y^{(1)}) = b \sum_{k=0}^{\infty} \sigma_k (h_k^4 - \lambda h_k^3) \delta_{h_k} \tag{10}$$

where we have denoted $h_k = kT + \tau_k$ and σ_k the jumps of u_a at h_k. Using the assumption $h_{k+1} - h_k > \Delta$, a "local" integration is now considered with $T(s) = (1 - e^{-\Delta s/3})/s$ instead of $H(s) = 1/s$. Here, $\mathrm{supp}\, T \subset (0, \Delta/3)$ and the coefficient 3 also represents the number of integrations required in order to ensure properness of the identification scheme. This results in

$$T^3 t^3 (t-\lambda)(y^{(3)} + a_1 y^{(2)} + a_0 y^{(1)}) \triangleq N - \lambda D$$

where, from the right hand side of (10), and by virtue of the support of a convolution product given in Section II-A, one has

$$\begin{aligned} &\mathrm{supp}\; T^3 \delta_{h_k} \subset (h_k, h_k + \Delta), \\ &\mathrm{supp}\,(N, D) \subset \{(h_k, h_k + \Delta) \mid k = 1, 2, \cdots\} \end{aligned}$$

On each of the latter intervals, the commutation instant h_k, and hence the delay τ_k are therefore obtained from the relation

$$\lambda = h_k = kT + \tau_k = N/D$$

As in the previous section, terms of the form $T^k t^l y^{(q)}$ involved in the expressions of N and D are implemented by considering first the development of $t^l y^{(q)}$ described in (2) and (3) and then the integration with T^k. A simulation result is given in Figure 5 with the parameters $a_0 = 2$, $a_1 = 1$, $b = 1$, $y(0) = 1.3$, $\dot{y}(0) = -2.3$, $T = 2$ and the delays $\{\tau_k\} = \{0.3, 0.5, 0.2, 0.3\}$. Note the same study could be conducted with unknown parameters a_0 and a_1 as well.

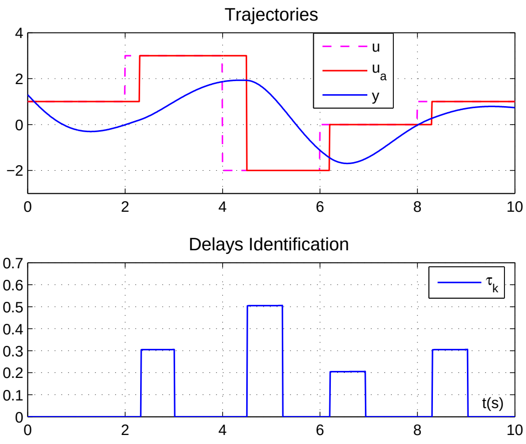

Fig. 5. Trajectories and delays identification

V. CONCLUSION

As in [5], [6], [8], [9], [10], the high speed convergence of our algorithms will permit to treat simultaneously the on-line identification and control of time-delay systems. Identifiability issues, joint estimations of delays and coefficients, multivariable systems with partial state measurements as well as the extension to discrete-time processes are under active investigations.

REFERENCES

[1] L. Belkoura, Identifiability of systems described by convolution equations, *Automatica*, vol. 41, 2005, pp. 505-512.

[2] L. Belkoura, M. Dambrine, Y. Orlov, J.-P. Richard, Identifiability and identification of linear systems with delays *Advances in Time-Delay Systems*, S.-I. Niculescu, K. Gu (Eds), Lect. Notes Comput. Sci. Engin., Springer, Berlin, vol. 38, 2004, pp. 123-135.

[3] F. Beltrán-Carvajal, G. Silva-Navarro, H. Sira-Ramírez, J. Quezada-Andrade, Active vibration control using on-line algebraic identification of harmonic vibrations, *Proc. Amer. Control Conf.*, Portland, OR, 2005.

[4] M. Fliess, Analyse non standard du bruit, *C.R. Acad. Sci. Paris, ser. I*, vol. 342, 2006, pp. 797-802.

[5] M. Fliess, C. Join, H. Sira-Ramírez, Robust residual generation for linear fault diagnosis: an algebraic setting with examples, *Internat. J. Control*, vol. 77, 2004, pp. 1223-1242.

[6] M. Fliess, C. Join, H. Sira-Ramírez, Closed-loop fault-tolerant control for uncertain nonlinear systems, *Control and Observer Design for Nonlinear Finite and Infinite Dimensional Systems*, T. Meurer, K. Graichen, E.D. Gilles (Eds), Lect. Notes Control Informat. Sci., Springer, Berlin, vol. 322, 2005, pp. 217-233.

[7] M. Fliess, R. Marquez, H. Mounier, An extension of predictive control, PID regulators and Smith predictors to some linear delay systems, *Internat. J. Control*, vol. 75, 2002, pp. 728-743.

[8] M. Fliess, M. Mboup, H. Mounier, H. Sira-Ramírez, Questioning some paradigms of signal processing via concrete examples, *Algebraic Methods in Flatness, Signal Processing and State Estimation*, H. Sira-Ramírez, G. Silva-Navarro (Eds.), Editiorial Lagares, México, 2003, pp. 1-21 (avalable: `http://hal.inria.fr/inria-00001059`).

[9] M. Fliess, H. Sira-Ramirez, An algebraic framework for linear identification, *ESAIM Contr. Optim. Cal. Variat.*, vol. 9, 2003, pp. 151-168.

[10] M. Fliess, H. Sira-Ramírez, Control via state estimations of some nonlinear systems, *Proc. Symp. Nonlinear Control Systems (NOLCOS 2004)*, Stuttgart, 2004 (avalable: `http://hal.inria.fr/inria-00001096`).

[11] X.M. Ren, A.B. Rad, P.T. Chan, W.L. Lo, On-line identification of continuous-time systems with unknown time-delay, *IEEE Trans. Automat. Contr.*, vol. 50, 2005, pp. 1418-1422.

[12] J.-P. Richard, Time-delay systems: an overview of some recent advances and open problems, *Automatica*, vol. 39, 2003, pp. 1667-1694.

[13] L. Schwartz, *Théorie des distributions* (2^{nd} ed.), Hermann, Paris, 1966.

[14] F. Veysset, L. Belkoura, P. Coton, J.-P. Richard, Delay system identification applied to the longitudinal flight of an aircraft through a vertical gust, *Proc.* 14^{th} *Medit. Conf. Control Automat.*, Ancona, Italy, 2006.